# 溶融樹脂の定常非等温非ニュートン純粘性多層流動に関する2.5D FEM 定式化

2.5D FEM formulation for
steady non-isothermal non-Newtonian
viscous multi-layer flow of polymer melt

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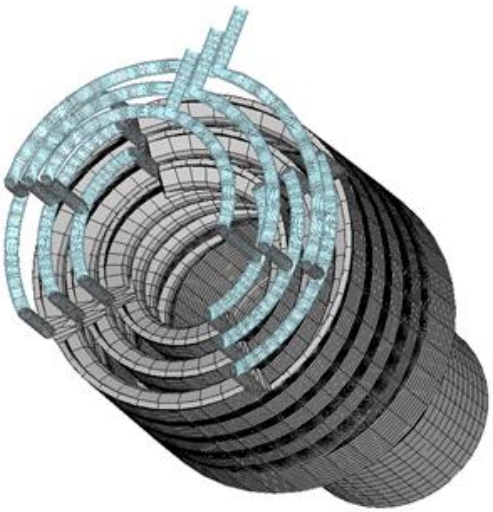
# ☐ Purpose

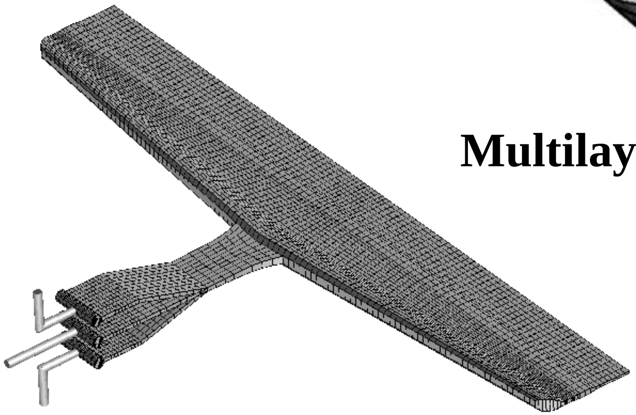
## Practical 2.5D FEM flow simulation for multilayer extrusion dies

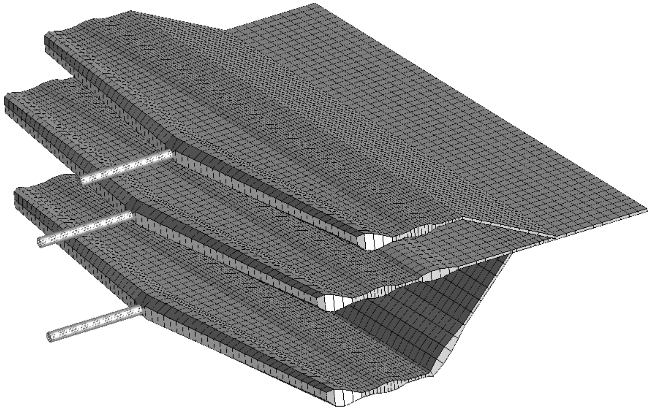
3D FEM: Large computational cost, difficult operation

2D FEM: Low resolution



Multilayer spiral mandrel die





Feed block type multilayer coat hanger die

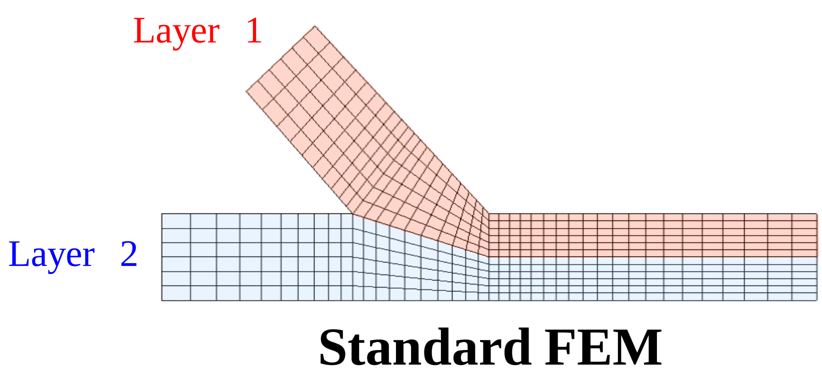
Multi-manifold coat hanger die

Fig. 1 Multi layer extrusion dies

# ☐ Merits of 2.5D FEM formulation



## ◇ Easy modelling



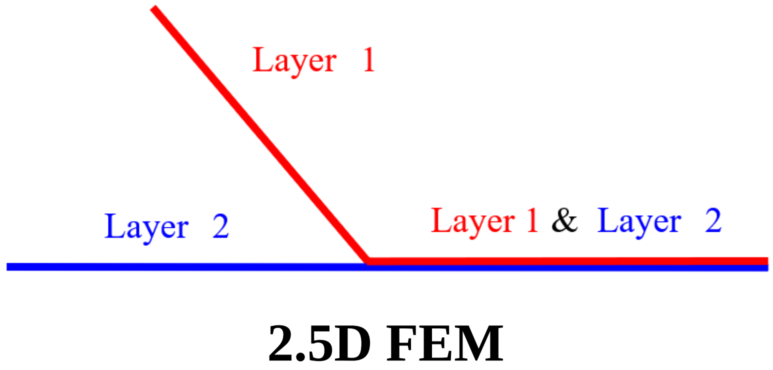




**Fig. 2 Analysis model of multi layer flow**

## ◇ Small computational storage

## ◇ Flexibility of shape representation by modification of thickness information

## ◇ High resolution solution in the thickness direction

# □ Break through of 2.5D FEM formulation



**Table 1 Boundary conditions on the interface of multi layer fluid I and II**

| | **Previous formulation** | **Present formulation** |
|---|---|---|
| Thermal flow field | **Fully developed state** | **Including under-developing state** |
| Velocity continuity | $\mathbf{V}^{I} = \mathbf{V}^{II}$ | |
| Shear stress continuity | $\eta^{I}\dot{\gamma}^{I} = \eta^{II}\dot{\gamma}^{II}$ | |
| Normal stress continuity | $\boxed{p^{I} = p^{II}}$ | $\boxed{-p^{I} + 2\eta^{I}\frac{1}{h^{I}}\frac{Dh^{I}}{Dt} = -p^{II} + 2\eta^{II}\frac{1}{h^{II}}\frac{Dh^{II}}{Dt}}$ |

($I$, $II$ : Layer number, $\mathbf{V}$ : tangential velocity vector,
$\eta$ : viscosity, $\dot{\gamma}$: shear strain rate, $p$:pressure, $h$:layer thickness)



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# □ Mathematical modeling of multilayer flow

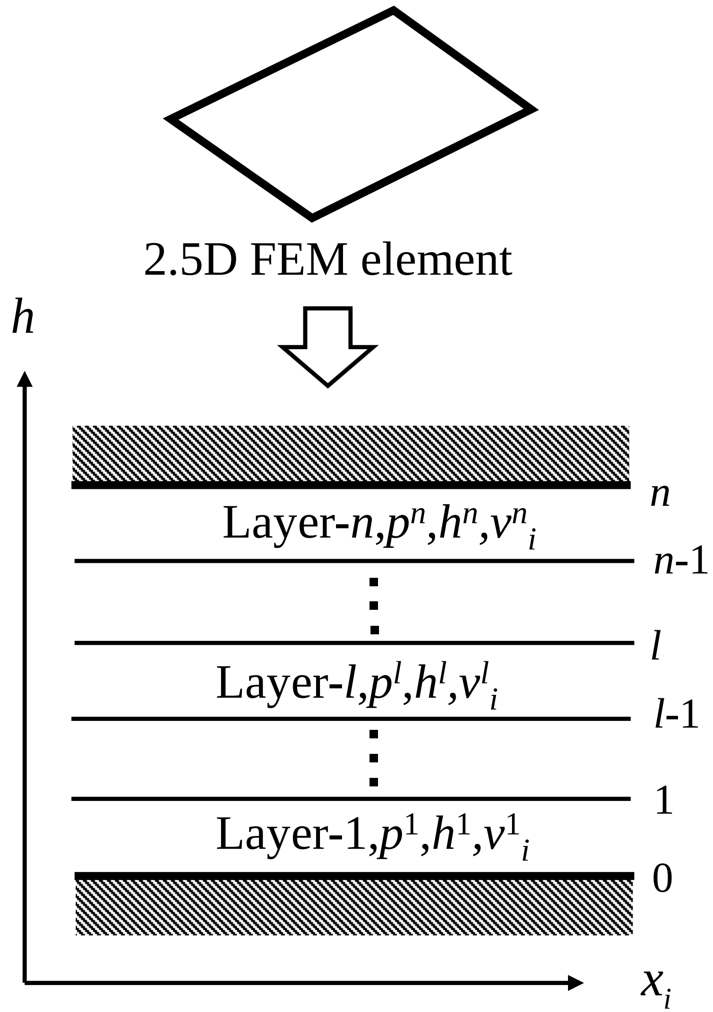



**Fig.3 Multilayer flow configuration (layer number :n)**

**Momentum equation**

$$\frac{\partial}{\partial h}\left(\eta^l \frac{\partial v_i^l}{\partial h}\right) = \frac{\partial p^l}{\partial x_i} \quad for\ 0 \le h \le h^l, l = 1 \sim n$$

**Shear stress**

$$\eta^l \frac{\partial v_i^l}{\partial h} = \frac{\partial p^l}{\partial x_i} h + A_i^l$$

**velocity**

$$v_i^l = \frac{\partial p^l}{\partial x_i} \int_0^h \frac{h}{\eta^l} dh + A_i^l \int_0^h \frac{1}{\eta^l} dh + B_i^l$$

$p^l$:pressure
$h^l$:thickness
$v^l_i$:velocity
$\eta^l$:viscosity
$A^l_i$,$B^l_i$ :integral constant
$i$=1,2,3

# Continuity conditions on the interface

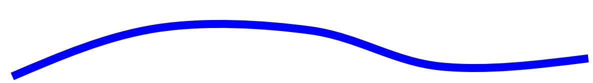
## Shear stress continuity

$$\eta^{l} \left.\frac{\partial v_i^{l}}{\partial h}\right|_{h=h^{l}} = \eta^{l+1} \left.\frac{\partial v_i^{l+1}}{\partial h}\right|_{h=0} \Rightarrow \boxed{\frac{\partial p^{l}}{\partial x_i} h^{l} + A_i^{l} = A_i^{l+1}}$$

## Velocity continuity

$$\left. v_i^{l} \right|_{h=h^{l}} = \left. v_i^{l+1} \right|_{h=0} \Rightarrow \boxed{\frac{\partial p^{l}}{\partial x_i} \int_0^{h^{l}} \frac{h}{\eta^{l}} dh + A_i^{l} \int_0^{h^{l}} \frac{1}{\eta^{l}} dh + B_i^{l} = B_i^{l+1}}$$

Layer-$l$+1,$p^{l+1}$,$h^{l+1}$,$v^{l+1}{}_i$



$l^{th}$ interface;$l$=1~$n$-1

Layer-$l$,$p^{l}$,$h^{l}$,$v^{l}{}_i$

**Fig.4 Interface of multilayer flow**

## Table 2 Simultaneous equations derived by the boundary conditions

| Condition | Equation number | Simultaneous conditional equations |
|---|---|---|
| Lower wall non-slip | 3 | $B_i^1 = 0$ |
| Velocity continuity on interface | 3(*n*-1) | $-A_i^l \alpha^l - B_i^l + B_i^{l+1} = \frac{\partial p^l}{\partial x_i} \beta^l$ |
| Shear stress continuity on interface | 3(*n*-1) | $-A_i^l + A_i^{l+1} = \frac{\partial p^l}{\partial x_i} h^l$ |
| Upper wall non-slip | 3 | $-A_i^n \alpha^n - B_i^n = \frac{\partial p^n}{\partial x_i} \beta^n$ |
| | Total 6n | |

$$\alpha^l = \int_0^{h^l} \frac{1}{\eta^l} dh, \beta^l = \int_0^{h^l} \frac{h}{\eta^l} dh$$

**Normal stress continuity**

$$-p^l + 2\eta^l \frac{1}{h^l}\mathbf{V}^l \bullet \nabla h^l = -p^{l+1} + 2\eta^{l+1}\frac{1}{h^{l+1}}\mathbf{V}^{l+1} \bullet \nabla h^{l+1} \quad for \quad l = 1 \sim n-1$$

**Constraint**

Element thickness : $H_e = \sum_{l=1}^{n} h^l$

**Energy equation**

$$\rho^l C_p^l \mathbf{V}^l \bullet \nabla T^l = \kappa^l \Delta T^l + \eta^l \left(\dot{\gamma}^l\right)^2 \quad for\ l = 1 \sim n$$

$T^l$:temperature $\rho^l$:density
$C^l_p$:heat capacity
$\kappa^l$:thermal conductivity

# SUPG(Streamline Upwind Petrov-Galerkin) FEM discretization

## Galerkin FEM discretization for continuity equation

$$\underbrace{S_{\alpha\beta}^{l} p_{\beta}^{l}}_{\text{Pressure gradient}} + \underbrace{Q_{\alpha}^{l}}_{\text{Flux}} + \underbrace{F_{\alpha}^{l}}_{\text{Layer interaction}} = 0 \quad for \ l = 1 \sim n$$

$$S_{\alpha\beta}^{l} = \gamma^{l} \int_{-1}^{1}\int_{-1}^{1} \frac{\partial \phi_{\alpha}}{\partial x_i} \frac{\partial \phi_{\beta}}{\partial x_i} J d\xi d\eta,$$

$$Q_{\alpha}^{l} = \int_{\Gamma_e} \phi_{\alpha} q_i^l n_i^l d\Gamma,$$

$$F_{\alpha}^{l} = \int_{-1}^{1}\int_{-1}^{1} \frac{\partial \phi_{\alpha}}{\partial x_i} f_i^{l} J d\xi d\eta$$

$$f_i^{l} = A_i^l \beta^l - h^l B_i^{l+1} \quad for \ \ l = 1 \sim n-1,$$

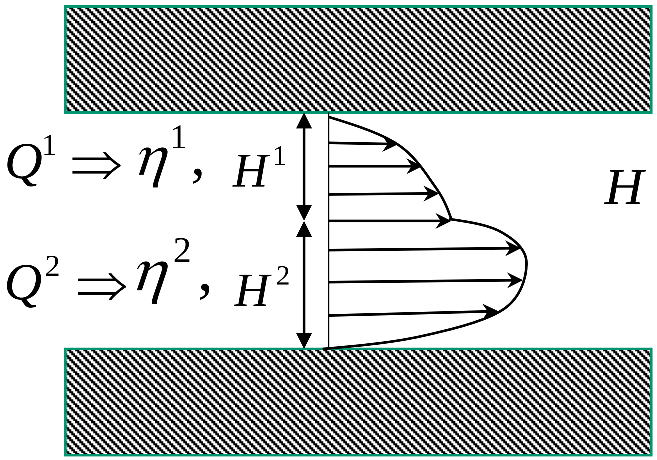
$$f_i^{n} = A_i^n \beta^n,$$

$$\gamma^{l} = \int_0^{h^l} \frac{(h)^2}{\eta^l} dh$$

$\alpha,\ \beta$: node number
$(\xi,\ \eta)$: local coordinate
$J$: Jacobian
$\phi$ : interpolation function

# ☐ Theoretical verification of 2.5D FEM formulation






$$H = H^1 + H^2$$

**Fig.5 Steady viscous flow between parallel plate**

$$\chi_h = \frac{H^2}{H^1}$$ : thickness ratio

$$\chi_q = \frac{Q^2}{Q^1}$$ : flux ratio

$$\chi_\eta = \frac{\eta^2}{\eta^1}$$ : viscous ratio

$$\boxed{\chi_h^4 = -4\chi_\eta\chi_h^3 - 3(\chi_\eta - \chi_q\chi_\eta)\chi_h^2 + 4\chi_q\chi_\eta\chi_h + \chi_q\chi_\eta^2}$$

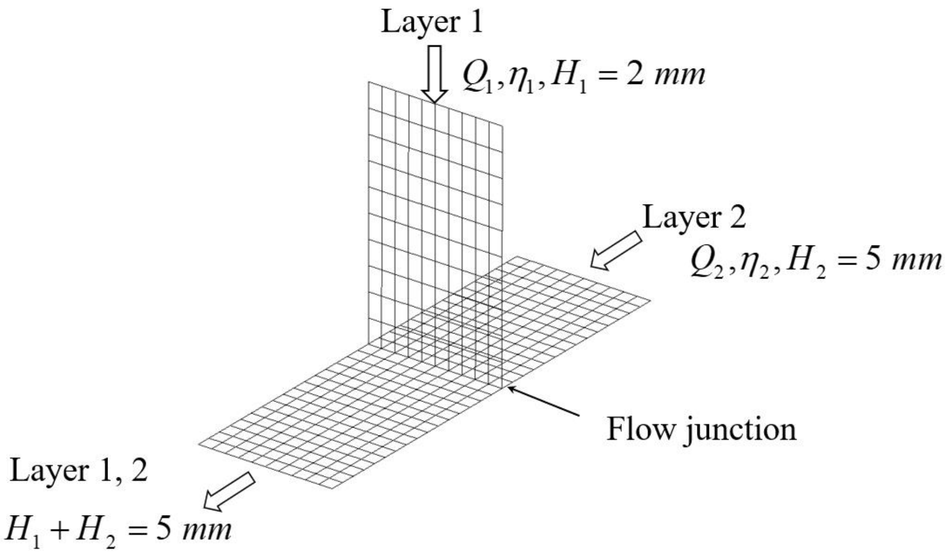



**Fig.6 Theoretical verification model**

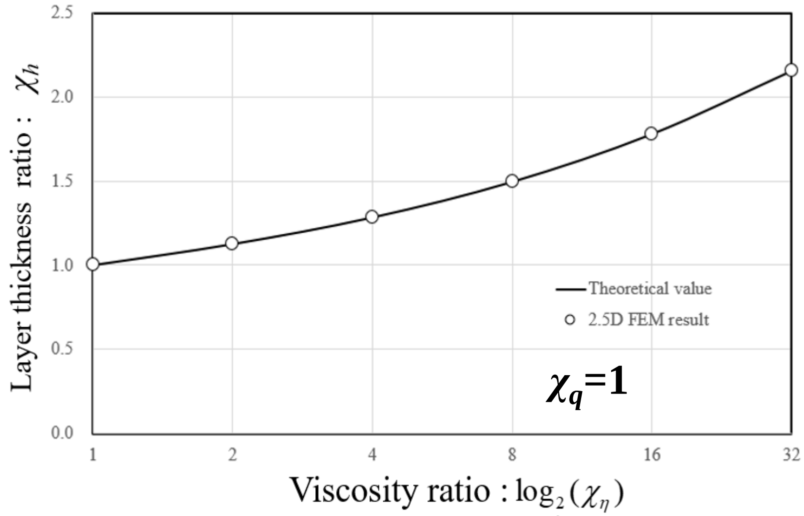



**Fig.7 Viscosity ratio dependency for thickness ratio**

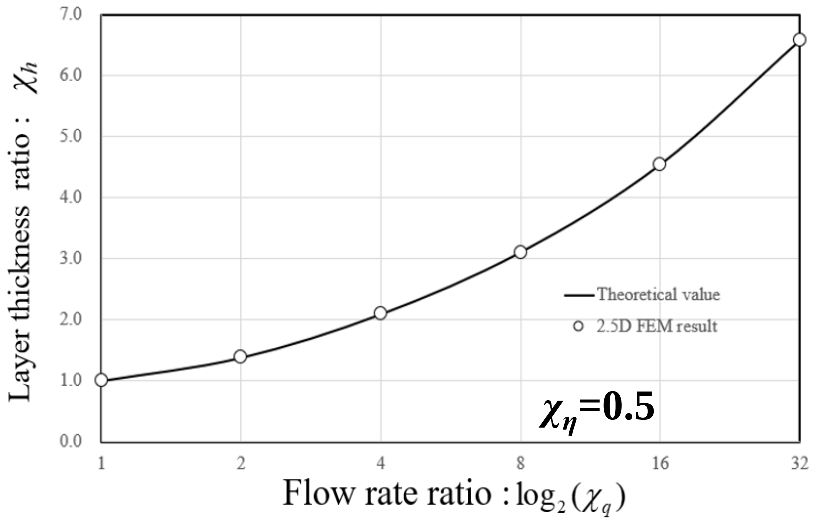



**Fig.8 Flow rate ratio dependency for thickness ratio**

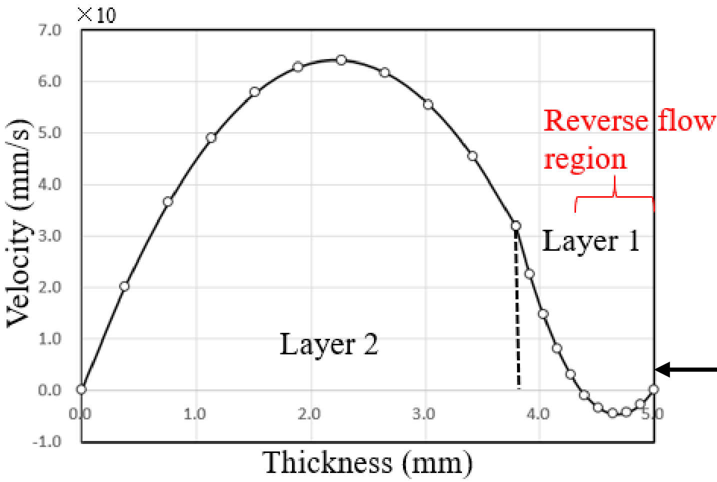
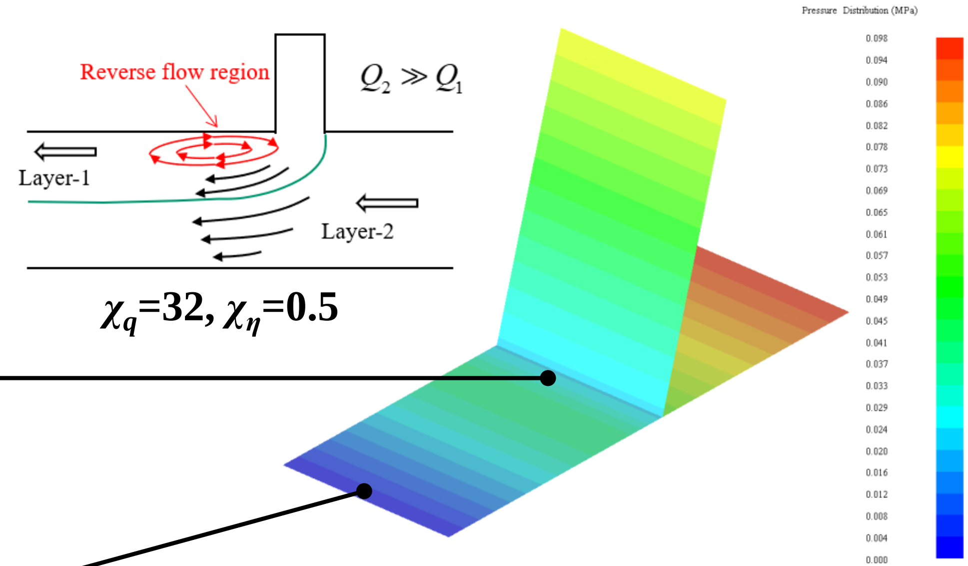



Fig.9 Velocity distribution near the multi layer flow junction



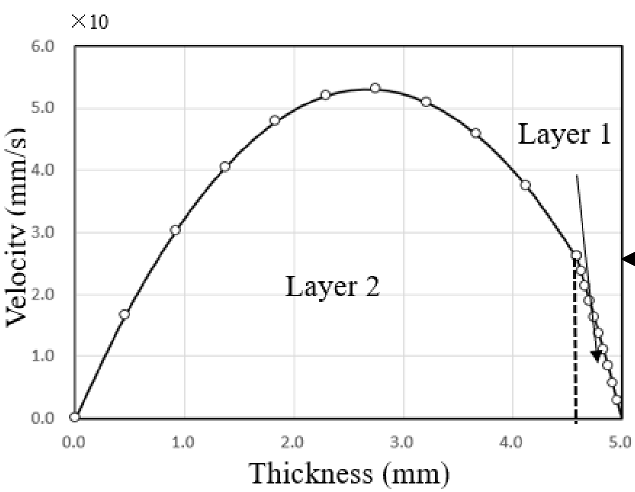




Fig.10 Velocity distribution at outlet

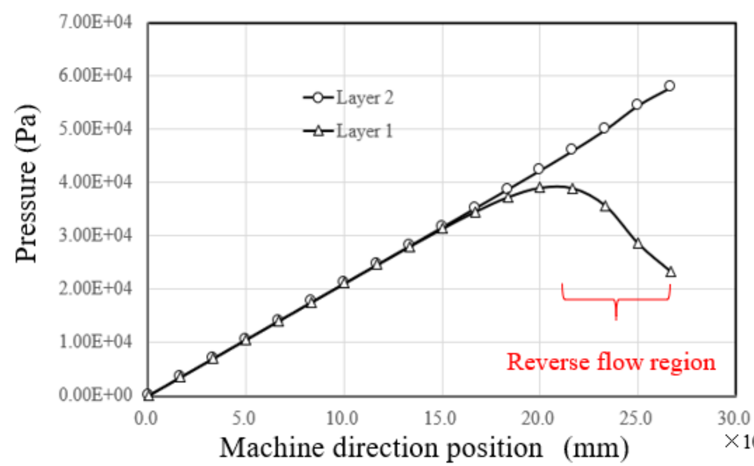
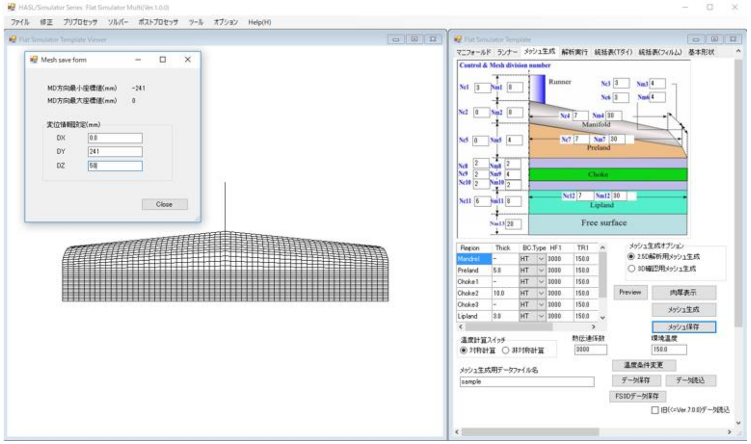



Fig.11 Pressure distribution along the machine direction

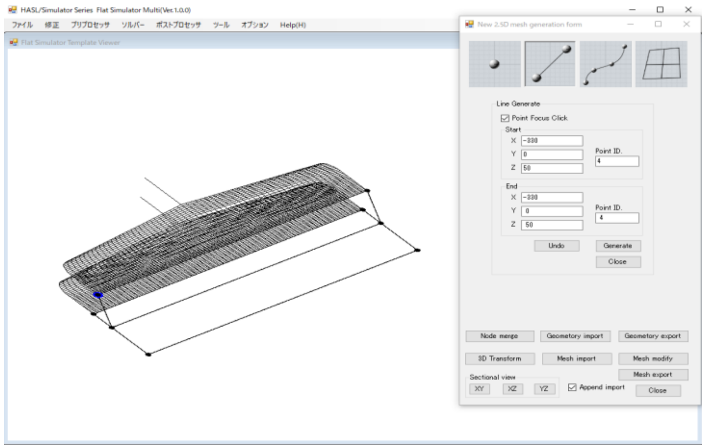
# □ Application of 2.5D FEM formulation



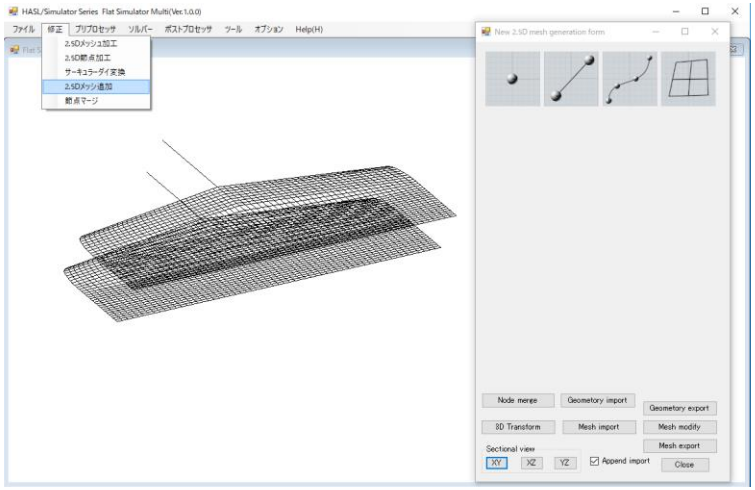
## Modeling of multi layer die



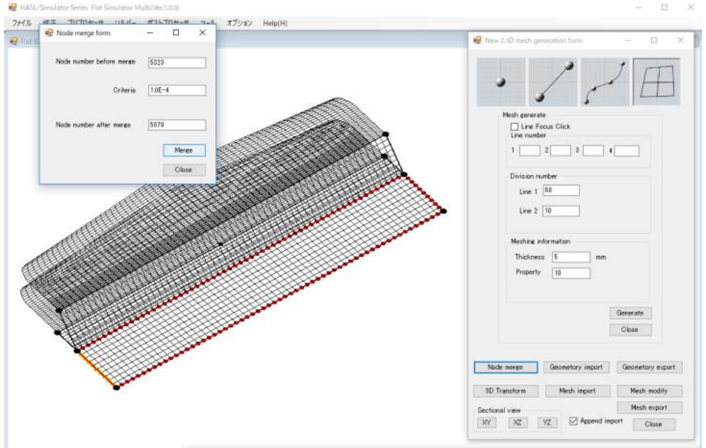
① Mesh generation for mono layer flow region using template



③ Geometrical representation for multi layer flow region

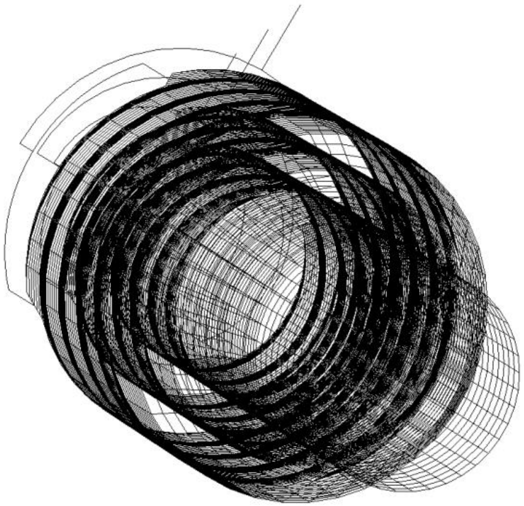


② Append import of mesh information

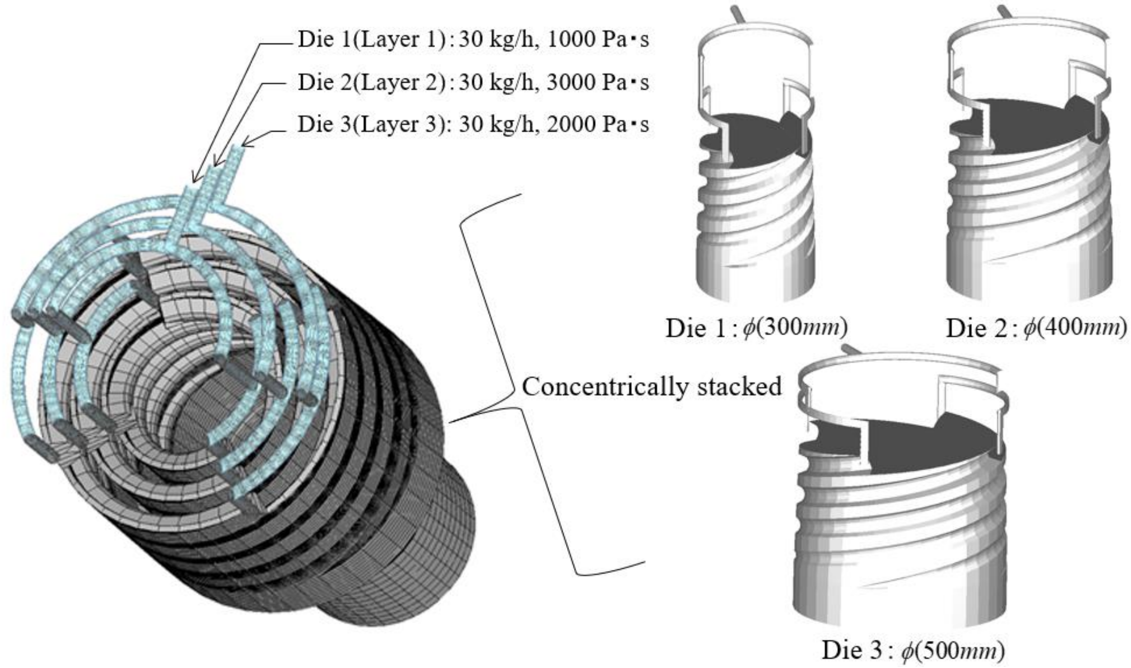


④ Mesh generation for multi layer flow region

# Test analysis
# (3-material 3-layer flow analysis in the spiral mandrel die)



2.5D FEM model




3D visualization model

**Fig. 12 Multilayer spiral mandrel die**

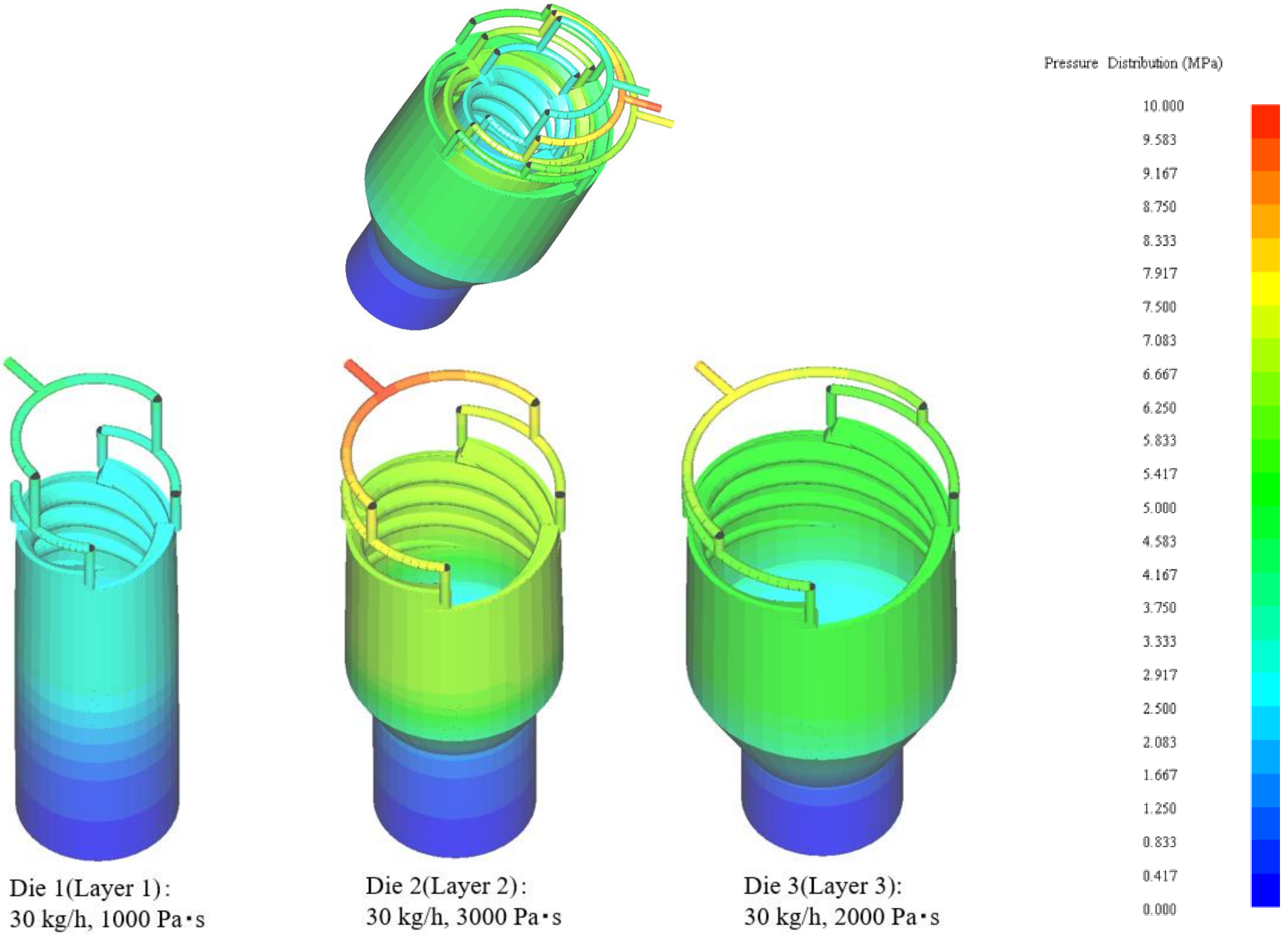



**Fig. 13 Pressure distribution**

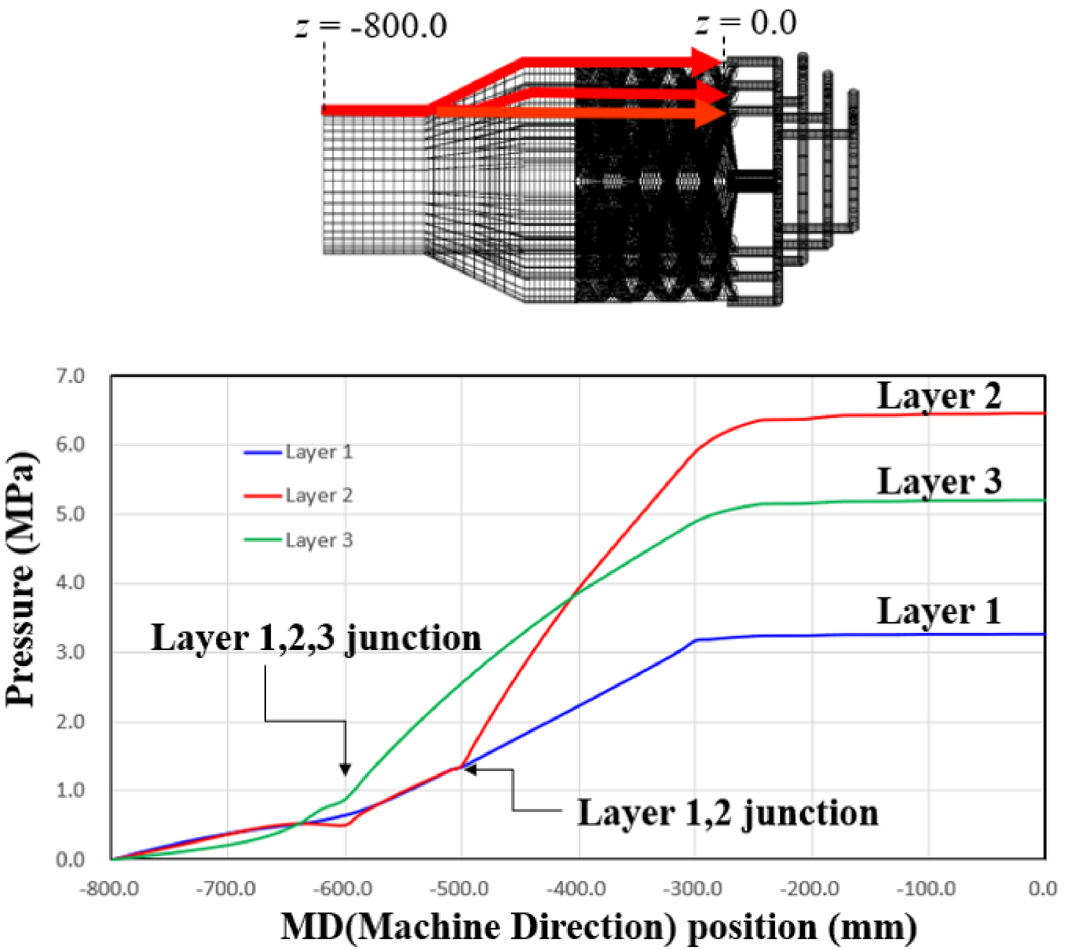



**Fig. 14 Pressure distribution in machine direction**

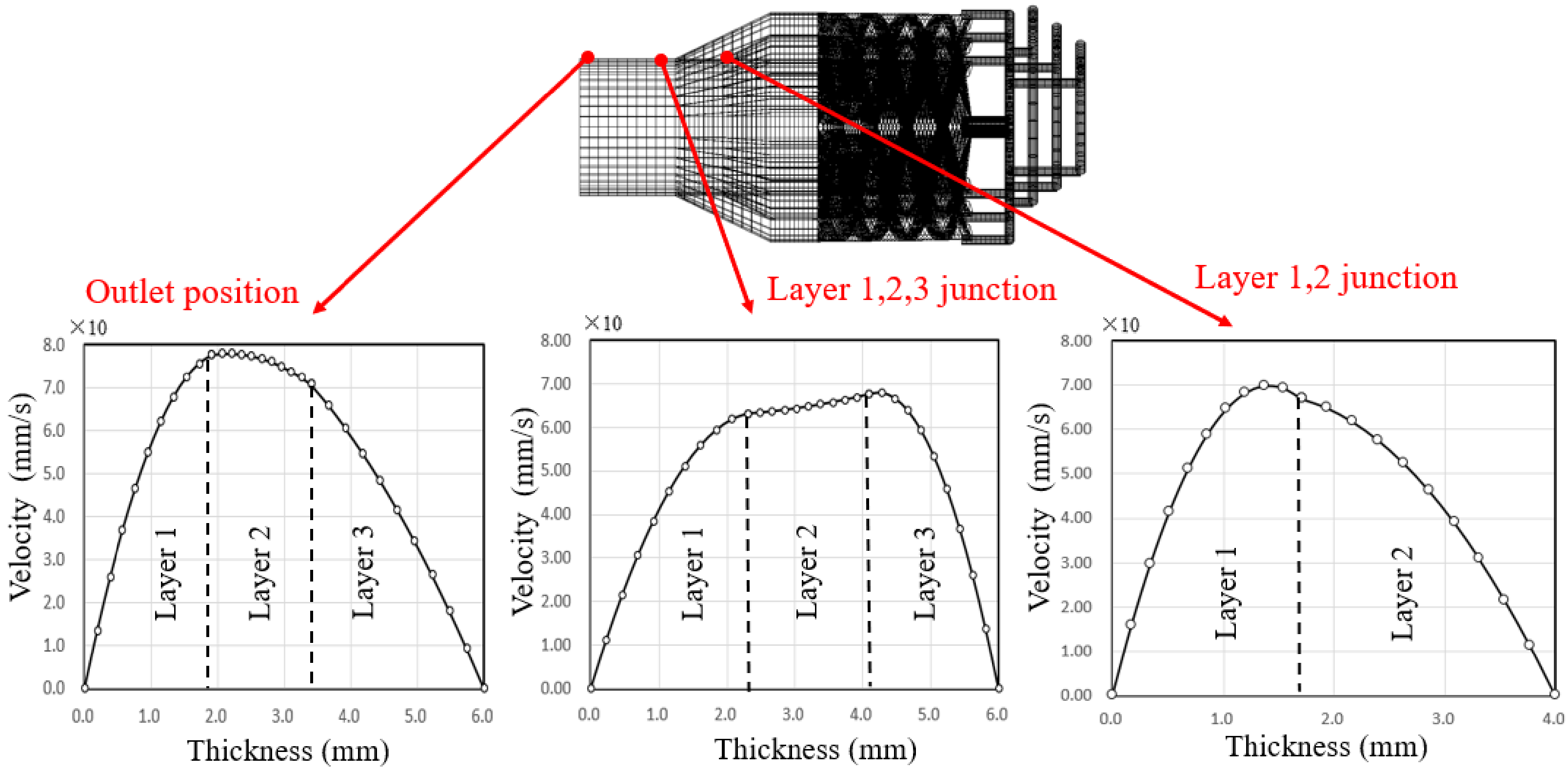
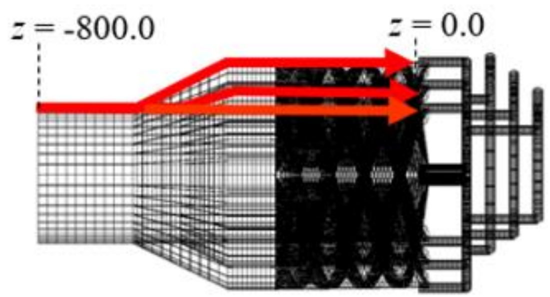



**Fig. 15 Velocity distribution in thickness direction**



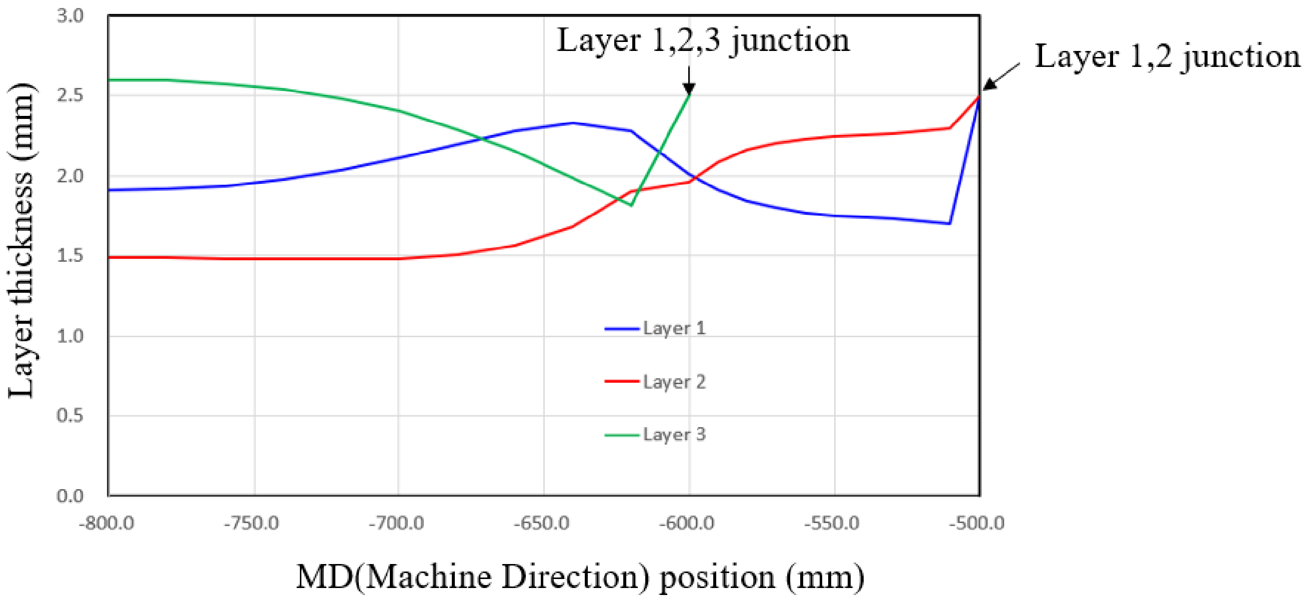




**Fig. 16 Thickness distribution in machine direction**

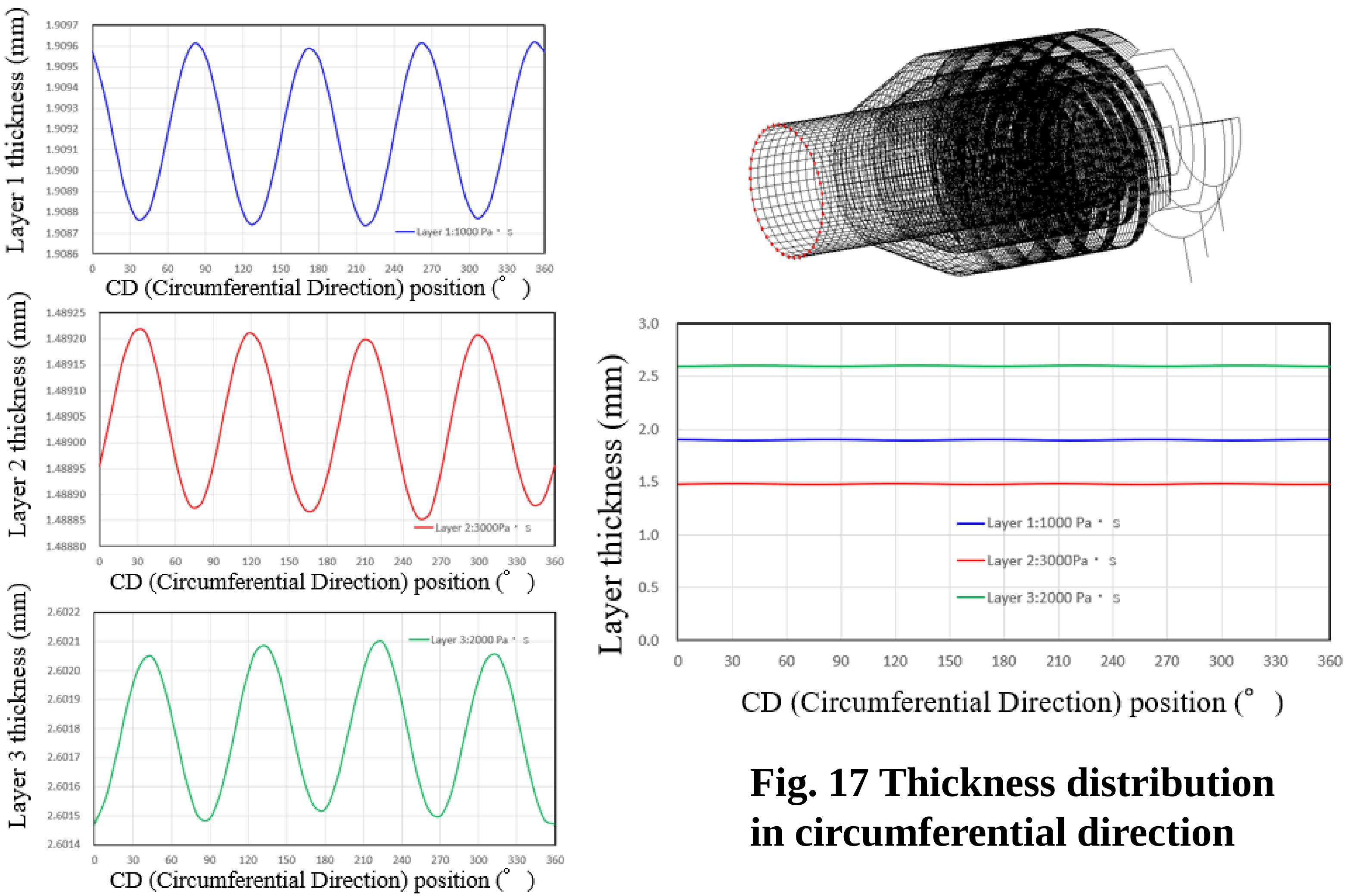



**Fig. 17 Thickness distribution in circumferential direction**

# □Conclusions



## 成果：

・Hele-Shaw 薄流れの定式化を一般化することで多層押出ダイ内の未発達状態を含む流動状況及び界面形成状態を効率的に評価可能な解析法を構築した。

・理論検証解析を通じて解析結果の妥当性を検討した。

## 今後の課題：

・検証解析の継続。

・2.5D FEM 粘弾性多層流動解析への定式化拡張。